EXTENDED ARMENDARIZ RINGS

NAZIM AGAYEV, ABDULLAH HARMANCI, AND SAIT HALICIOGLU

ABSTRACT. In this note we introduce central linear Armendariz rings as a generalization of Armendariz rings and investigate their properties.

AMS Subject Classification: 16U80

Key words: reduced rings, central reduced rings, abelian rings, Armendariz rings, linear Armendariz rings, central linear Armendariz rings.

1. INTRODUCTION

Throughout this paper R denotes an associative ring with identity. Rege and Chhawchharia [13], introduce the notion of an Armendariz ring. The ring R is called *Armendariz* if for any $f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{j=0}^{s} b_j x^j \in R[x]$, $f(x)g(x) = 0$ implies $a_i b_j = 0$ for all i and j. The name of the ring was given due to Armendariz who proved that reduced rings (i.e. rings without nonzero nilpotent elements) satisfied this condition [2].

Number of papers have been written on the Armendariz rings (see, e.g. [1], [9]). So far, Armendariz rings are generalized in different ways (see namely, [6], [12]). In particular, Lee and Wong [10] introduced *weak Armendariz rings* (i.e. if the product of two linear polynomials in $R[X]$ is 0, then each product of their coefficients is 0), Liu and Zhao [12] introduce also *weak Armendariz rings* (if the product of two polynomials in $R[X]$ is 0, then each product of their coefficients is nilpotent) as another generalization of Armendariz rings. To get rid of confusion, we call the rings *linear Armendariz* which satisfy Lee and Wong condition. A ring R is called *central linear Armendariz*, if the product of two linear polynomials in $R[X]$ is 0, then each product of their coefficients is central. Clearly, Armendariz rings are linear Armendariz and linear Armendariz rings are central linear Armendariz. In case R is reduced ring every weak Armendariz ring is central linear Armendariz. We supply some examples to show that the converses of these statements need not be true in general. We prove that the class of central linear Armendariz rings lies strictly between classes of linear Armendariz rings and abelian rings. For a ring R, it is shown that the polynomial ring $R[x]$ is central linear Armendariz if and only if the Laurent polynomial ring $R[x,x^{-1}]$ is central linear Armendariz. Among others we also show that R is reduced ring if and only if the matrix ring $T_n^k(R)$ is Armendariz ring if and only if the matrix ring $T_n^{n-2}(R)$ is central linear Armendariz ring, for a natural number $n \geq 3$ and $k = [n/2]$. And for an ideal I of R, if R/I central linear Armendariz and I is reduced, then R is central linear Armendariz.

We also introduce central reduced rings as a generalization of reduced rings. The ring R is called *central reduced* if every nilpotent is central. We prove that if R is central reduced ring, then R is central linear Armendariz, and if R is central reduced ring, then the trivial extension $T(R,R)$ is central linear Armendariz. Moreover, it is proven that if R is a semiprime ring, then R is central reduced ring if and only if $R[x]/(x^n)$ is central linear Armendariz, where $n \geq 2$ is a natural number and (x^n) is the ideal generated by x^n.

We write $R[x], R[[x]], R[x,x^{-1}]$ and $R[[x,x^{-1}]]$ for the polynomial ring, the power series ring, the Laurent polynomial ring and the Laurent power series ring over R, respectively.

2. Central Linear Armendariz Rings

In this section central linear Armendariz rings are introduced as a generalization of linear Armendariz rings. We prove that some results of linear Armendariz rings can be extended to central linear Armendariz rings for this general settings. Clearly, every Armendariz ring is linear Armendariz. However, linear Armendariz rings are not necessarily Armendariz in general (see [10, Example 3.2]).

We now give a possible generalization of linear Armendariz rings.

Definition 2.1. *The ring R is called central linear Armendariz if the product of two linear polynomials in $R[X]$ is 0, then each product of their coefficients is central.*

Note that all commutative rings, reduced rings, Armendariz rings and linear Armendariz rings are central linear Armendariz. It is clear that subrings of central linear Armendariz rings are central linear Armendariz.

Recall that R is said to be *abelian* if idempotent elements of R are central.

Lemma 2.2. *If the ring R is central linear Armendariz, then R is abelian.*

Proof. Let e be any idempotent in R, consider $f(x) = e - er(1-e)x, g(x) = (1-e) + er(1-e)x \in R[x]$ for any $r \in R$. Then $f(x)g(x) = 0$. By hypothesis, in particular $er(1-e)$ is central. Therefore $er(1-e) = 0$. Hence $er = ere$ for all $r \in R$. Similarly we consider

$h(x) = (1-e)-(1-e)rex$ and $t(x) = e+(1-e)rex$ in $R[x]$ for any $r \in R$. Then $h(x)t(x) = 0$. As before $(1-e)re = 0$ and $ere = re$ for all $r \in R$. It follows that e is central element of R, that is, R is abelian. □

Example 2.3. *Let R be any ring. For any integer $n \geq 2$, consider the ring $R^{n\times n}$ of $n \times n$ matrices and the ring $T_n(R)$ of $n \times n$ upper triangular matrices over R. The rings $R^{n\times n}$ and $T_n(R)$ contain non-central idempotents. Therefore they are not abelian. By Lemma 2.2 these rings are not central linear Armendariz.*

Recall that a ring R is *semicommutative*, if for any $a,b \in R$, $ab = 0$ implies $aRb = 0$.

Theorem 2.4. *Let R be a von Neumann regular ring R. Then the following are equivalent:*
(1) *R is Armendariz.*
(2) *R is reduced.*
(3) *R is central linear Armendariz.*
(4) *R is linear Armendariz.*
(5) *R is semicommutative.*

Proof. By Lemma 2.2 and [5, Lemma 3.1, Theorem 3.2], we have $(3) \Rightarrow (2)$. $(2) \Rightarrow (5)$ Clear. $(5) \Rightarrow (2)$ Let $a^2 = 0$ for $a \in R$. By (5), $aRa = 0$. So $(aR)^2 = 0$. Assume $aR \neq 0$. By hypothesis, aR contains a non-zero idempotent. This is a contradiction. Hence $a = 0$. The rest is clear from [1, Theorem 6]. □

We now give a condition for a ring to be central linear Armendariz relating to central idempotents.

Lemma 2.5. *Let R be a ring and e an idempotent of R. If e is a central idempotent of R, then the following are equivalent:*
(1) *R is central linear Armendariz.*
(2) *eR and $(1-e)R$ are central linear Armendariz.*

Proof. (1) $\Rightarrow$ (2) Since the subrings of central linear Armendariz rings are central linear Armendariz, (2) holds.
(2) $\Rightarrow$ (1) Let $f(x) = a_0 + a_1x$, $g(x) = b_0 + b_1x$ be non zero polynomials in $R[x]$. Assume that $f(x)g(x) = 0$. Let $f_1 = ef(x)$, $f_2 = (1-e)f(x)$, $g_1 = eg(x)$, $g_2 = (1-e)g(x)$. Then $f_1(x)g_1(x) = 0$ in $(eR)[x]$ and $f_2(x)g_2(x) = 0$ in $((1-e)R)[x]$. By (2) ea_ieb_j is central in eR and $(1-e)a_i(1-e)b_j$ is central in $(1-e)R$ for all $0 \leq i \leq 1$, $0 \leq j \leq 1$. Since e and

$1-e$ central in R, $R = eR \oplus (1-e)R$ and so $a_i b_j = ea_i b_j + (1-e)a_i b_j$ is central in R for all $0 \leq i \leq 1$, $0 \leq j \leq 1$. Then R is central linear Armendariz. $\square$

Clearly, any linear Armendariz ring is central linear Armendariz. We now prove that the converse is true if the ring is right $p.p.-$ring.

Theorem 2.6. *If the ring R is linear Armendariz, then R is central linear Armendariz. The converse holds if R is right $p.p.-$ring.*

Proof. Suppose R is central linear Armendariz and right $p.p.-$ring. Let $f(x) = a_0 + a_1 x$, $g(x) = b_0 + b_1 x \ \in R[x]$. Assume $f(x)g(x) = 0$ Then we have:

$$\begin{aligned} a_0 b_0 &= 0 && (1) \\ a_0 b_1 + a_1 b_0 &= 0 && (2) \\ a_1 b_1 &= 0 && (3) \end{aligned}$$

By hypothesis there exist idempotents $e_i \in R$ such that $r(a_i) = e_i R$ for all i. So $b_0 = e_0 b_0$ and $a_0 e_0 = 0$. Multiply (2) from the right by e_0, by Lemma 2.2, R is abelian and we have $0 = a_0 b_1 e_0 + a_1 b_0 e_0 = a_0 e_0 b_1 + a_1 b_0 e_0 = a_1 b_0$. So $a_0 b_1 = 0$. Hence R is linear Armendariz. This completes the proof. $\square$

Let R be a ring and let M be an (R,R)-bimodule. The *trivial extension* of R by M is defined to be the ring $T(R,M) = R \oplus M$ with the usual addition and the multiplication $(r_1, m_1)(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2)$.

Example 2.7 shows that the assumption ”right p.p.-ring” in Theorem 2.6 is not superfluous.

Example 2.7. *There exists a central linear Armendariz ring which is neither right p.p.-ring nor linear Armendariz ring.*

Proof. Let n be an integer with $n \geq 2$. Consider the ring $R = T(\mathbf{Z}_{2^n}, \mathbf{Z}_{2^n})$. If $a = 2^{n-1}$ and $f(x) = \begin{bmatrix} \bar{a} & \bar{0} \\ \bar{0} & \bar{a} \end{bmatrix} + \begin{bmatrix} \bar{a} & \bar{1} \\ \bar{0} & \bar{a} \end{bmatrix} x \in R[x]$, then $(f(x))^2 = 0$. Because $\begin{bmatrix} \bar{a} & \bar{0} \\ \bar{0} & \bar{a} \end{bmatrix} \begin{bmatrix} \bar{a} & \bar{1} \\ \bar{0} & \bar{a} \end{bmatrix} \neq 0$, R is not a linear Armendariz ring. Since R is commutative, it is central linear Armendariz ring. Moreover, since the principal ideal $I = \begin{bmatrix} 0 & \mathbf{Z}_{2^n} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} R$ is not projective, R is not right p.p.-ring. $\square$

Now we will introduce a notation for some subrings of $T_n(R)$. Let k be a natural number smaller than n. Say

$$T_n^k(R) = \left\{ \sum_{i=j}^{n} \sum_{j=1}^{k} a_j e_{(i-j+1)i} + \sum_{i=j}^{n-k} \sum_{j=1}^{n-k} r_{ij} e_{j(k+i)} : a_j, r_{ij} \in R \right\}$$

where e_{ij}' s are matrix units. Elements of $T_n^k(R)$ are in the form

$$\begin{bmatrix} x_1 & x_2 & \dots & x_k & a_{1(k+1)} & a_{1(k+2)} & \cdots & a_{1n} \\ 0 & x_1 & \dots & x_{k-1} & x_k & a_{2(k+2)} & \cdots & a_{2n} \\ 0 & 0 & x_1 & \dots & & & & a_{3n} \\ & & & \dots & & & & \\ & & & & & & & x_1 \end{bmatrix}$$

where $x_i, a_{js} \in R$, $1 \le i \le k$, $1 \le j \le n-k$ and $k+1 \le s \le n$.

For a reduced ring R, our aim is to investigate necessary and sufficent conditions for $S = T_n^k(R)$ to be central linear Armendariz. In [11], Lee and Zhou prove that, if R is reduced ring, then S is Armendariz ring for $k = [n/2]$. Hence S is linear Armendariz and so S is central linear Armendariz. In the following, we show that the converse of this theorem is also true. Moreover, it is proven that R is reduced ring if and only if $T_n^k(R)$ is Armendariz ring if and only if $T_n^{n-2}(R)$ is central linear Armendariz ring. In this direction, we need the following lemma:

Lemma 2.8. *Suppose that there exist $a,b \in R$ such that $a^2 = b^2 = 0$ and $ab = ba$ is not central. Then R is not a central linear Armendariz ring.*

Proof. $(a+bx)(a-bx) = 0$ in $R[x]$, but ab is not central. So, R is not a central linear Armendariz ring. □

Theorem 2.9. *Let $n \ge 3$ be a natural number. Then R is reduced ring if and only if $T_n^k(R)$ is central linear Armendariz ring, where $1 \le k \le n-2$.*

Proof. Let R be a reduced ring. In [11], it is shown that $T_n^k(R)$ is Armendariz ring and so it is central linear Armendariz. Conversely, suppose that R is not a reduced ring. Choose a nonzero element $a \in R$ with square zero. Then for elements $A = a(e_{11}+e_{22}+\dots+e_{nn}), B = e_{1(k+1)} + e_{1(k+2)} + \dots + e_{1n}$ in $T_n^k(R)$, $A^2 = B^2 = 0$ and $AB = BA$ is not central, since $(AB)(e_{1(n-k)} + e_{2(n-k+1)} + \dots + e_{k(n-1)} + e_{(k+1)n}) = ae_{1n} \neq 0$. Therefore, from Lemma 2.8, $T_n^k(R)$ is not central linear Armendariz ring. This completes the proof. □

Theorem 2.10. *Let R be a ring, $n \ge 3$ be a natural number and $k = [n/2]$. Then the following are equivalent:*

(1) *R is reduced ring.*

(2) *$T_n^k(R)$ is Armendariz ring.*

(3) *$T_n^{n-2}(R)$ is central linear Armendariz ring.*

Proof. $(1) \Rightarrow (2)$ See [11].

$(2) \Rightarrow (3)$ Since subrings of Armendariz rings are Armendariz, the rest is clear.

$(3) \Rightarrow (1)$ It follows from Theorem 2.9. □

Note that the homomorphic image of a central linear Armendariz ring need not be central linear Armendariz. If R is commutative and Gaussian ring, by [1, Theorem 8] every homomorphic image of R is Armendariz and so it is central linear Armendariz.

In [7], it was shown that for a ring R, if I is a reduced ideal of R such that R/I is Armendariz, then R is Armendariz. For central linear Armendariz rings we have the similar result.

Theorem 2.11. *Let R/I be central linear Armendariz and I be reduced. Then R is central linear Armendariz.*

Proof. Let $a, b \in R$. If $ab = 0$, then $(bIa)^2 = 0$. Since $bIa \subseteq I$ and I is reduced, $bIa = 0$. Also, $(aIb)^3 \subseteq (aIb)(I)(aIb) = 0$. Therefore $aIb = 0$. Assume $f(x) = a_0 + a_1x, g(x) = b_0 + b_1x \in R[x]$ and $f(x)g(x) = 0$. Then

$$
\begin{aligned}
a_0b_0 &= 0 \qquad (1)\\
a_0b_1 + a_1b_0 &= 0 \qquad (2)\\
a_1b_1 &= 0 \qquad (3)
\end{aligned}
$$

We first show that for any a_ib_j, $a_iIb_j = b_jIa_i = 0$. Multiply (2) from the right by Ib_0, we have $a_1b_0Ib_0 = 0$, since $a_0b_1Ib_0 = 0$. Then $(b_0Ia_1)^3 \subseteq b_0I(a_1b_0Ia_1b_0)Ia_1 = 0$. Hence $b_0Ia_1 = 0$. This implies $a_1Ib_0 = 0$. Multiply (2) from the left by a_0I, we have $a_0Ia_0b_1 + a_0Ia_1b_0 = 0$ and so $a_0Ia_0b_1 = 0$. Thus $(b_1Ia_0)^3 = 0$ and $b_1Ia_0 = 0$. Therefore $a_0Ib_1 = 0$. Since R/I is central Armendariz, it follows that $\overline{a_i}\overline{b_j}$ is central in R/I. So $a_ib_jr - ra_ib_j \in I$ for any $r \in R$. Now from above results, it can be easily seen that $(a_ib_jr - ra_ib_j)I(a_ib_jr - ra_ib_j) = 0$. Then $a_ib_jr = ra_ib_j$ for all $r \in R$. Hence a_ib_j is central for all i and j. This completes the proof.

□

Let S denote a multiplicatively closed subset of R consisting of central regular elements. Let $S^{-1}R$ be the localization of R at S. Then we have:

Proposition 2.12. *R is central linear Armendariz if and only if $S^{-1}R$ is central linear Armendariz.*

Proof. Suppose that R is a central linear Armendariz ring. Let $f(x)=\sum_{i=0}^{1}(a_i/s_i)x^i$, $g(x)=\sum_{j=0}^{1}(b_j/t_j)x^j \in (S^{-1}R)[x]$ and $f(x)g(x)=0$. Then we may find u, v, c_i and d_j in S such that $uf(x)=\sum_{i=0}^{1}a_ic_ix^i \in R[x]$, $vg(x)=\sum_{i=0}^{1}b_jd_jx^j \in R[x]$ and $(uf(x))(vg(x))=0$. By supposition $(a_ic_i)(b_jd_j)$ are central in R for all i and j. Since c_i and d_j are regular central elements of R, a_ib_j are central in R for all i and j. It follows that $(a_i/s_i)(b_j/t_j)$ are central for all i and j. Conversely, assume that $S^{-1}R$ is a central linear Armendariz ring. Let $f(x)=\sum_{i=0}^{1}a_ix^i, g(x)=\sum_{j=0}^{1}b_jx^j \in R[x]$. Assume $f(x)g(x)=0$. Then $f(x)/1=\sum_{i=0}^{1}(a_i/1)x^i, g(x)=\sum_{j=0}^{1}(b_j/1)x^j \in S^{-1}R[x]$ and $(f(x)/1)(g(x)/1)=0$ in $S^{-1}R$. By assumption $(a_i/1)(b_j/1)$ is central in $S^{-1}R$. Hence, for all i and j, a_ib_j is central in R. $\square$

Corollary 2.13. *For any ring R, the polynomial ring $R[x]$ is central linear Armendariz if and only if the Laurent polynomial ring $R[x,x^{-1}]$ is central linear Armendariz.*

Proof. Let $S=\{1,x,x^2,x^3,x^4,\ldots\}$. Then S is a multiplicatively closed subset of $R[x]$ consisting of central regular elements. Then the proof follows from Proposition 2.12. $\square$

We now define *central reduced rings* as a generalization of reduced rings.

Definition 2.14. *The ring R is called central reduced ring if every nilpotent element is central.*

Example 2.15. *All commutative rings, all reduced rings and all strongly regular rings are central reduced.*

One may suspect that central reduced rings are reduced. But the following example erases the possibility.

Example 2.16. *Let S be a commutative ring and $R=S[x]/(x^2)$. Then R is commutative ring and so R is central reduced. If $a=x+(x^2)\in R$, then $a^2=0$. Therefore R is not a reduced ring.*

It is well known that if the ring R is reduced, then R is linear Armendariz. In our case, we have the following:

Theorem 2.17. *If R is central reduced ring, then R is central linear Armendariz.*

Proof. Let $f(x) = a_0 + a_1x$, $g(x) = b_0 + b_1x \in R[x]$. Assume $f(x)g(x) = 0$. Then we have :

$$\begin{aligned} a_0b_0 &= 0 && (1)\\ a_0b_1 + a_1b_0 &= 0 && (2)\\ a_1b_1 &= 0 && (3)\end{aligned}$$

Since $(b_0a_0)^2 = 0$ and $(b_1a_1)^2 = 0$, $b_0a_0, b_1a_1 \in C(R)$, where $C(R)$ is the center of R. Multiply (2) from the right by a_0, we have $a_0b_1a_0 + a_1b_0a_0 = 0$. Thus $a_0b_1a_0 + b_0a_0a_1 = 0$. Multiply last equation from the left by a_0, we have $a_0{}^2b_1a_0 = 0$ and so $(a_0b_1a_0)^2 = 0$, that is, $a_0b_1a_0 \in C(R)$. Hence $(a_0b_1)^3 = 0$ and so $a_0b_1 \in C(R)$. Similarly it can be shown that $a_1b_0 \in C(R)$. □

Note that if R is reduced ring, by [13, Proposition 2.5] trivial extension $T(R,R)$ is Armendariz and so it is linear Armendariz. For central reduced rings, we have

Lemma 2.18. *If R is central reduced ring, then the trivial extension $T(R,R)$ is central linear Armendariz. The converse holds if R is semiprime.*

Proof. Let $f(x) = \begin{bmatrix} a_0 & b_0 \\ 0 & a_0 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 \\ 0 & a_1 \end{bmatrix} x = \begin{bmatrix} f_1(x) & f_2(x) \\ 0 & f_1(x) \end{bmatrix}$, $g(x) = \begin{bmatrix} c_0 & d_0 \\ 0 & c_0 \end{bmatrix} + \begin{bmatrix} c_1 & d_1 \\ 0 & c_1 \end{bmatrix} x = \begin{bmatrix} g_1(x) & g_2(x) \\ 0 & g_1(x) \end{bmatrix} \in T(R,R)[x]$. If $f(x)g(x) = 0$, then we have

$$f(x)g(x) = \begin{bmatrix} f_1(x)g_1(x) & f_1(x)g_2(x) + f_2(x)g_1(x) \\ 0 & f_1(x)g_1(x) \end{bmatrix} = 0.$$

Hence $f_1(x)g_1(x) = 0$, $f_1(x)g_2(x) + f_2(x)g_1(x) = 0$. In this case, we have

$$\begin{aligned} a_0c_0 &= 0 && (1)\\ a_0c_1 + a_1c_0 &= 0 && (2)\\ a_1c_1 &= 0 && (3)\end{aligned}$$

From (1) and (3), $a_0c_0, a_1c_1 \in C(R)$ and so $c_0a_0, c_1a_1 \in C(R)$. Multiply (2) from the right by a_0, we have $a_0c_1a_0 + a_1c_0a_0 = 0$. Thus $a_0c_1a_0 + c_0a_0a_1 = 0$, so $a_0{}^2c_1a_0 = 0$ and so $(a_0c_1a_0)^2 = 0$, that is, $a_0c_1a_0 \in C(R)$. Hence $(a_0c_1)^3 = 0$ and so $a_0c_1 \in C(R)$. Similarly it can be shown that $a_1c_0 \in C(R)$.

Conversely, suppose R is semiprime and $S = T(R,R)$ is central linear Armendariz. Let

$a^n = 0$ with $a \in R$. Consider

$$f(x) = \begin{bmatrix} a^{n-1} & 0 \\ 0 & a^{n-1} \end{bmatrix} + \begin{bmatrix} a^{n-1} & 1 \\ 0 & a^{n-1} \end{bmatrix} x,$$

$g(x) = \begin{bmatrix} a^{n-1} & 0 \\ 0 & a^{n-1} \end{bmatrix} + \begin{bmatrix} a^{n-1} & -1 \\ 0 & a^{n-1} \end{bmatrix} x \in S[x]$. Then $f(x)g(x) = 0$. Hence $\begin{bmatrix} 0 & a^{n-1} \\ 0 & 0 \end{bmatrix} \in C(S)$ and so $a^{n-1} \in C(R)$. Therefore $(a^{n-1}R)^2 = 0$ implies $a^{n-1} = 0$. Continuing in this way, we have $a = 0$. □

In [1, Theorem 5], Anderson and Camillo proved that for a ring R and $n \geq 2$ a natural number, $T_n^{n-1}(R)$ is Armendariz if and only if R is reduced. Lee and Wong [10, Theorem 3.1] also proved that $T_n^{n-1}(R)$ is linear Armendariz if and only if R is reduced. For central linear Armendariz rings, we have the following.

Theorem 2.19. *Let R be a semiprime ring and n ≥ 2 a natural number. R is central reduced ring if and only if $T_n^{n-1}(R)$ is central linear Armendariz.*

Proof. Suppose R is central reduced ring. Let $a^2 = 0$ for $a \in R$. Then $a \in C(R)$ and so $aRa = 0$. Since R is semiprime, we have $a = 0$. Therefore R is reduced and $T_n^{n-1}(R)$ is Armendariz by [1, Theorem 5]. Hence $T_n^{n-1}(R)$ is linear Armendariz and by Theorem 2.6, it is central linear Armendariz. Conversely, assume that $T_n^{n-1}(R)$ is central linear Armendariz. Using the similar technique as in the proof of Lemma 2.18, it can be shown that R is central reduced. □

References

[1] D.D. Anderson and V. Camillo, *Armendariz rings and Gaussian rings*, Comm. Algebra 26(7)(1998), 2265-2272.

[2] E. Armendariz, *A note on extensions of Baer and p.p.-rings*, J. Austral. Math. Soc. 18(1974), 470-473.

[3] G.F. Birkenmeier, J. Y. Kim and J. K. Park, *Principally quasi-Baer rings*, Comm. Algebra 29(2)(2001), 639-660.

[4] E.W. Clark, *Twisted matrix units semigroup algebras*, Duke Math. J. 34(1967), 417-424.

[5] K.R. Goodearl, *Von Neumann Regular Rings*, second edition, Krieger Publishing Co., Malabar, Florida, 1991.

[6] C.Y. Hong, N.K. Kim and T.K. Kwak, *On skew Armendariz rings*, Comm. Algebra 31(1)(2003), 103-122.

[7] C. Huh, Y. Lee and A. Smoktunowicz, *Armendariz rings and semicommutative rings*, Comm. Algebra 30(2)(2002), 751-761.

[8] I. Kaplansky, *Rings of operators*, W. A. Benjamin, New York, 1968.

[9] N.K. Kim and Y. Lee, *Armendariz rings and reduced rings*, J. Algebra 223(2000), 477-488.

[10] T.K. Lee and T.L. Wong, *On Armendariz rings*, Houston J. Math. 29 (2003), 583-593.

[11] T.K. Lee and Y. Zhou, *Armendariz and reduced rings*, Comm. Algebra 32(6)(2004), 2287-2299.

[12] L. Liu and R. Zhao, *On weak Armendariz rings*, Comm. Algebra 34(7)(2006), 2607-2616.

[13] M.B. Rege and S.Chhawchharia, *Armendariz rings*, Proc. Japan Acad. Ser. A Math. Sci. 73(1997), 14-17.

NAZIM AGAYEV, QAFQAZ UNIVERSITY, DEPARTMENT OF PEDAGOGY, BAKU, AZERBAIJAN

E-mail address: `nazimagayev@qafqaz.edu.az`

ABDULLAH HARMANCI, HACETTEPE UNIVERSITY, DEPARTMENT OF MATHEMATICS, ANKARA TURKEY

E-mail address: `harmanci@hacettepe.edu.tr`

SAIT HALICIOGLU, DEPARTMENT OF MATHEMATICS, ANKARA UNIVERSITY, 06100 ANKARA, TURKEY

E-mail address: `halici@ankara.edu.tr`